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Efficiency of cosmic ray reflections from an ultrarelativistic shock wave

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ABSTRACT

The process of cosmic ray acceleration up to energies in excess of 10^{20} eV at relativistic shock waves with large Lorentz factors, $\Gamma \gg 1$ requires $\sim \Gamma^2$ particle energy gains at single reflections from the shock (cf. Gallant & Achterberg 1999). In the present comment, applying numerical simulations we address an efficiency problem arising for such models. The actual efficiency of the acceleration process is expected to be substantially lower than the estimates of previous authors.

Key words: acceleration of particles – UHE cosmic rays – gamma ray bursts – shock waves

1 INTRODUCTION

In attempt to substitute a single question mark for the previous two, some authors try to identify the process accelerating particles to ultra high energies ('UHE', energy $E > 10^{18}$ eV) with ultrarelativistic shock waves considered to be sources of γ-ray bursts (cf. Waxman 1995a,b, Vietri 1995, Milgrom & Usov 1995, Gallant & Achterberg 1999, ≡ GA99). In the proposed models, reaching cosmic ray energies in excess of 10^{20} eV requires that particles are reflected from the shock wave characterized with a large Lorentz gamma factor, Γ, to enable a relative energy gain – in a single reflection – comparable to Γ^2 (cf. GA99). On the other hand, basing on our experience with numerical modeling, we suggested (Bednarz & Ostrowski 1998, Ostrowski 1999) that such processes cannot actually work due to low efficiency of particle reflections.

In the present note we elaborate this problem in detail with the use of numerical modeling of particle interactions with the shock. We show that 'Γ^2' reflections can occur with a non-negligible rate only if a substantial amount of turbulence is present downstream of the shock. However, even in such conditions the number of accelerated particles is a small fraction of all cosmic rays hitting the shock.

2 SIMULATIONS

As discussed by Bednarz & Ostrowski (1998), and in detail by GA99, particles accelerated in multiple interactions with an ultrarelativistic shock wave gain on average in a single 'loop' – upstream-downstream-upstream – the amount of energy comparable to the original energy, $< \Delta E > \approx E$. It is due to extreme particle anisotropy occurring in large Γ shocks. Particles hitting the shock wave the first time, with their isotropic upstream distribution, can receive higher energy gains. In this case an individual reflection from the shock may increase particle energies on a factor of $\sim \Gamma^2$. However the effectiveness of such acceleration depends on how many of particles from the original upstream population can be reflected. To verify it we use Monte Carlo simulations similar to the applied earlier to derivation of the accelerated particle spectra (Bednarz & Ostrowski 1998, for details see also Bednarz & Ostrowski 1996). The code reproduces perturbations of particle trajectories due to MHD turbulence by applying discrete scatterings of particle direction within a narrow cone along its momentum vector. A procedure uses a hybrid approach involving very small scattering angles close to the shock and larger angles further away from it. Between successive scatterings particle trajectories are derived in the uniform background magnetic field. The respective scaling of the time between the successive scattering acts close and far from the shock mimics the same turbulence amplitude everywhere. The scattering amplitude was selected in a way to reproduce a pitch angle diffusion process for particle momentum. It requires the angular scattering amplitude of particle momentum vector to be much smaller than the particle anisotropy. All computations were done in a respective local plasma rest frame and the Lorentz transformation was applied to every particle crossing of the shock.

Particles with initial momenta p_0 taken as a momentum unit, $p_0 = 1$, were injected at the distance of $2r_g$ (r_g - particle gyroradius) upstream of the shock front. For all such particles we derived their trajectories until they crossed the shock upstream, or were caught in the downstream plasma, reaching a distance of $4r_g$ downstream of the shock. For each single particle interaction with the shock the particle momentum vector was recorded, so we were able to consider

angular and energy distributions of such particles. We considered shocks with Lorentz factors $\Gamma = 10$, 160 and 320. For each shock we discussed the acceleration processes in conditions with the magnetic field inclination $\psi = 0°$, $10°$, $70°$ and with 16 values for the turbulence amplitude measured by the ratio τ of the cross-field diffusion coefficient, $\kappa_\perp$, to the parallel diffusion coefficient, $\kappa_\parallel$. The applied values of τ were taken from the range of $(3.2 \cdot 10^{-6},\ 0.95)$, approximately uniformly distributed in $\log\tau$. In each simulation run we derived trajectories of $5{\cdot}10^4$ particles with initial momenta isotropically distributed in the upstream rest frame.

3 EFFICIENCY OF 'Γ^2' REFLECTIONS

In the downstream plasma rest frame the ultrarelativistic shock moves with velocity $c/3$. This velocity is comparable to the particle velocity c. Therefore, from all particles crossing the shock downstream only the ones with particular momentum orientations will interact with the shock again, the remaining particles will be caught in the downstream plasma flow and advected far from the shock front. In the simulations we considered this process quantitatively. However, let us first present a simple illustration.

Large compression ratios occurring in ultrarelativistic shocks, as measured between the upstream and downstream plasma rest frames, lead for nearly all oblique upstream magnetic field configurations to the quasi-perpendicular configurations downstream of the shock. Thus, let us consider for this illustrative example a shock with a non-perturbed perpendicular downstream magnetic field distribution. Particle crossing the shock downstream with inclination to the magnetic field θ and the phase ϕ – both measured in the downstream plasma rest frame, $\phi = \pi/2$ for particles normal to the shock and directed downstream – will be able to cross the shock upstream only if the equation

$$\frac{c}{3}\,t = r_{\rm g}\left[\cos(\phi + \omega_{\rm g} t) - \cos\phi\right] \qquad (3.1)$$

has a solution at positive time t. Here $r_{\rm g} = \frac{pc}{eB}\sin\theta$ is the particle gyroradius, $\omega_{\rm g} = \frac{eB}{p}$ is the gyration frequency, and other symbols have the usual meaning.

An angular range in the space (θ, ϕ) enabling particles crossing the shock downstream to reach the shock again can be characterized for illustration by three values of θ. Particles with $\sin\theta = 1$ are able to reach the shock again if $\phi \in(1.96,\ 3.48)$, with $\sin\theta = 0.5$ if $\phi \in(2.96,\ 3.87)$ and with $\sin\theta = 1/3$ only for $\phi = 4.71$. That means that all particles with ϕ smaller than 1.96 (Fig. 1) are not able to reach the shock again if fluctuations of the magnetic field downstream of the shock are not present.

For perturbed magnetic fields some downstream trajectories starting in the (θ, ϕ) plane outside the reflection range can be scattered toward the shock to cross it upstream. We prove it by simulations presented in Fig. 2. One may observe that increasing the perturbation amplitude leads to increased number of reflected particles, reaching $\approx 13\%$ in the limit of $\tau = 1$. For large magnetic field fluctuations the mean relative energy gains of reflected particles are close to $1.2\Gamma^2$ for the shock Lorentz factors considered. One may note that for small ψ and Γ the energy gain increases with growing τ. The points resulting from simulations for the

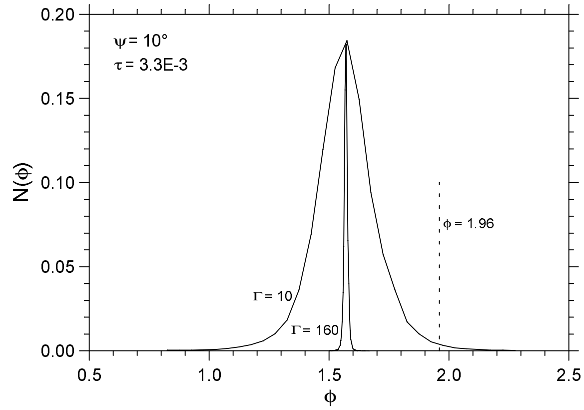

Figure 1. A distribution of particle phases for particles crossing the shock downstream (as measured in the downstream plasma rest frame), if their upstream distribution was isotropic. A dashed line delimits a range of particle phases below which particles are not able to reach the shock again at the perpendicular uniform downstream magnetic field.

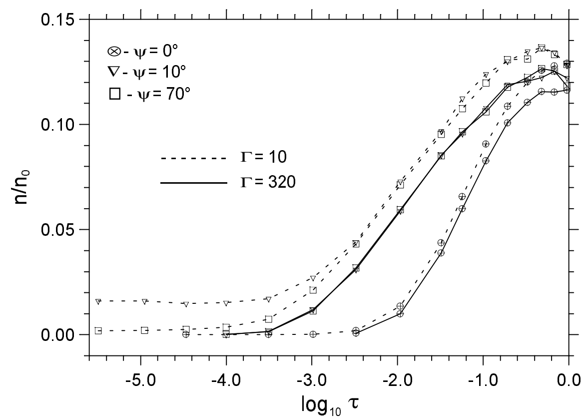

Figure 2. A ratio of the number of reflected particles, n, to all particles crossing the shock downstream, n_0, as a function of the magnetic field fluctuations amplitude, τ.

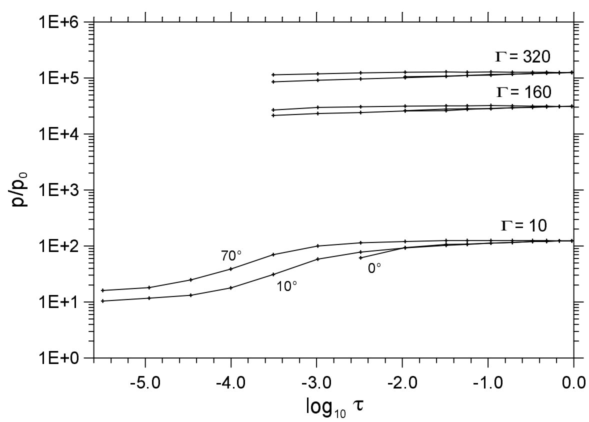

Figure 3. Momentum gains of reflected particles, p/p_0, as a function of the magnetic field fluctuations amplitude, τ. For large magnetic field fluctuations the momentum gain approaches $\approx 1.2\Gamma^2$ independently of the shock Lorentz factor.

smallest values of τ were not included into Fig. 3 because of small number of reflected particles (cf. Fig. 2).

4 SUMMARY

We have shown that efficiency of 'Γ^2' reflections in ultra-relativistic shock waves strongly depends on fluctuations of magnetic field downstream of the shock. In the most favorable conditions with high amplitude turbulence downstream the shock the reflection efficiency is a factor of 10 or more smaller than the values assumed by other authors. Moreover, due to the magnetic field compression at the shock, we do not expect the required large values of $\kappa_\perp/\kappa_\parallel$ to occur behind the shock (cf. a different approach of Medvedev & Loeb 1999). Therefore, with the actual efficiency of 1 - 10 % there is an additional difficulty for models postulating UHE particle acceleration at GRB shocks (cf. GA99). Let us note, however, that the mean downstream trajectory of the reflected particle involves only a fraction of its gyroperiod. Thus the presence of compressive long waves in this region, leading to non-random trajectory perturbations could modify our estimates.

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